



REVIEW OF THE SELECTION AND PERFORMANCE EVALUATION PROCEDURES FOR JAW CRUSHERS

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Abstract

Crushing and grinding of rocks and other particles has many important applications, including coarse crushing mined ore and quarry rock, fine grinding of coal for power station boilers, and for production of paint, ceramics, cement and other materials. Crushing and grinding of rocks and other particles are governed by the laws of physics involving mass, velocity, kinetic energy, gravity, etc. As crushing and grinding activities evolves in the mining industry, the issue of optimization becomes paramount. Crushing plants operate under very harsh conditions and in most cases involve very abrasive materials. Multiple factors therefore influence the performance of crushers.

This paper examines the procedure for selecting and evaluating the performance of jaw crushers. It is however important to note that it is impractical to reduce the process of selecting and sizing a crusher to a series of formulas. The selection process is largely based on experience and testing with actual field applications and laboratory tests that show how a given material will be reduced by a given crusher type. The paper examines the critical design parameters of jaw crushers. Crushers encounter considerable difficulties concerning external loads acting on their mechanical joints which make it almost impracticable to use theoretical calculations for design and performance evaluation. It is therefore necessary to use experimental studies to verify any theoretical methods used for these purposes. It should be noted however that considering the variety of applications as well as rock materials, minerals and ores, a truly optimal performance of a crushing application must be based on an optimized crusher design as well as a continuously optimized crusher operation.

1.0 Introduction

Jaw crushers are one of the most commonly used crushers in the world. They can easily be distinguished by the presence of two plates. One plate is fixed where the other opens and closes. This process ensures that the materials are trapped and crushed. Three types of jaw crushers are presently widely used. They are the Blake, Dodge, and Universal jaw crushers. The classification is based on the location of the pivot point of the swinging jaw. The Blake crusher is most commonly used of the three and it was designed and patented by Eli Whitney Blake in 1858 (Weiss, 1985). Jaw crushers have found wide use in quarries, mining, ore crushing and metallurgical plants, where the material physical condition varies from dry to slightly wet, not sticky. The single toggle jaw crusher has found wide used because of its higher capacity and much lower cost when compare to other types of jaw crushers (Donovan, 2003). Jaw crushers are very suitable for rocks that has the hardness index of medium-hard to very-hard and so are very common in quarries that requires the primary crushing of blasted rockpile. They can be applied to rocks or ore with edge lengths of more than 2,000 mm (Donovan, 2003).

2.0 Description of the Double Toggle and the Single Toggle Jaw Crusher

A noted characteristic of single toggle jaw crushers is that the swing jaw is suspended on the eccentric drive shaft (Figure 1) and the toggle plate that is placed against the crusher frame supports the swing jaw. The whole arrangement results in a motion that results in a higher capacity than that achieved by the double-toggle jaw crushers (Figure 2). The characteristics of different feeds have influenced the variation in design of the single-toggle jaw crushers by rock hardness.

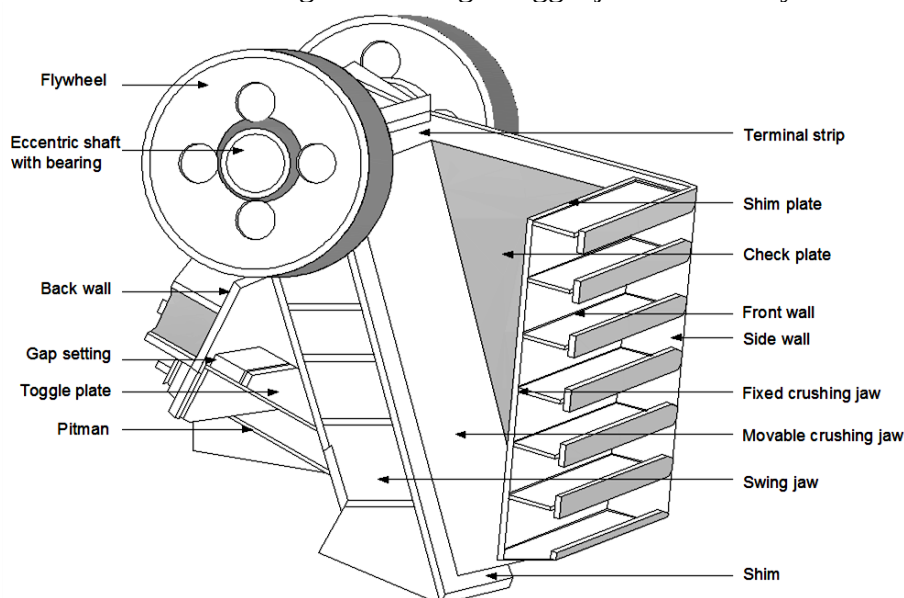


Figure 1. The various parts of a single-toggle jaw crusher (*adapted from www.thyssenkrupp-industrial-solutions.com/backenbrecher_en.pdf*)

Double-toggle jaw crushers are fitted with a system of toggle levers which upward and downward movement is effected by a pitman by means of an eccentric shaft (Figure 3). The resultant effect of the upward and downward movement is the oscillating motion of the swing jaw. The ores or rocks to be crushed are fed to the crusher by allowing them fall by gravity and the necessary pressure to crush the material is achieved when the crushing chamber is narrowed by the movement of the swing jaw. To increase the crushing effect, the crushing jaw lining is toothed. The retraction

of the swing jaw allows the crushed materials to leave the crusher. The crusher is designed in such a way that as crushed materials exits the crusher, new uncrushed materials enter concurrently (Figure 4). The transmission of the required power to the relevant parts of the crusher is effected by the toggle lever system which makes this type of crusher the equipment of choice for the crushing of very hard, toughest and abrasive materials.

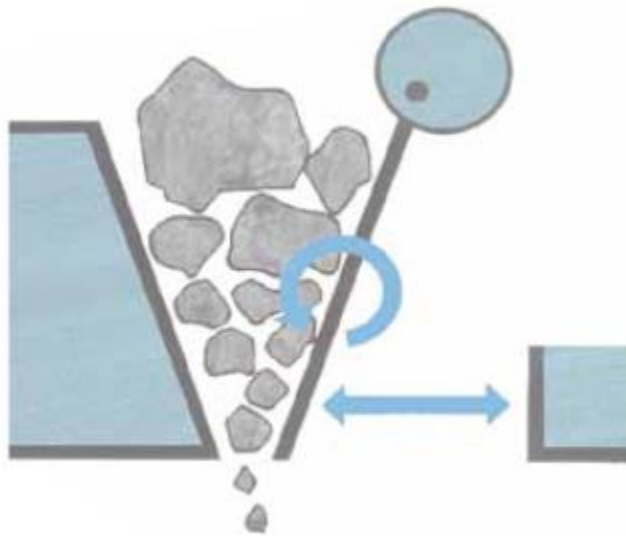


Figure 2. How a single-toggle jaw crusher with the toggle system operates (www.thyssenkrupp-industrial-solutions.com/backenbrecher_en.pdf)

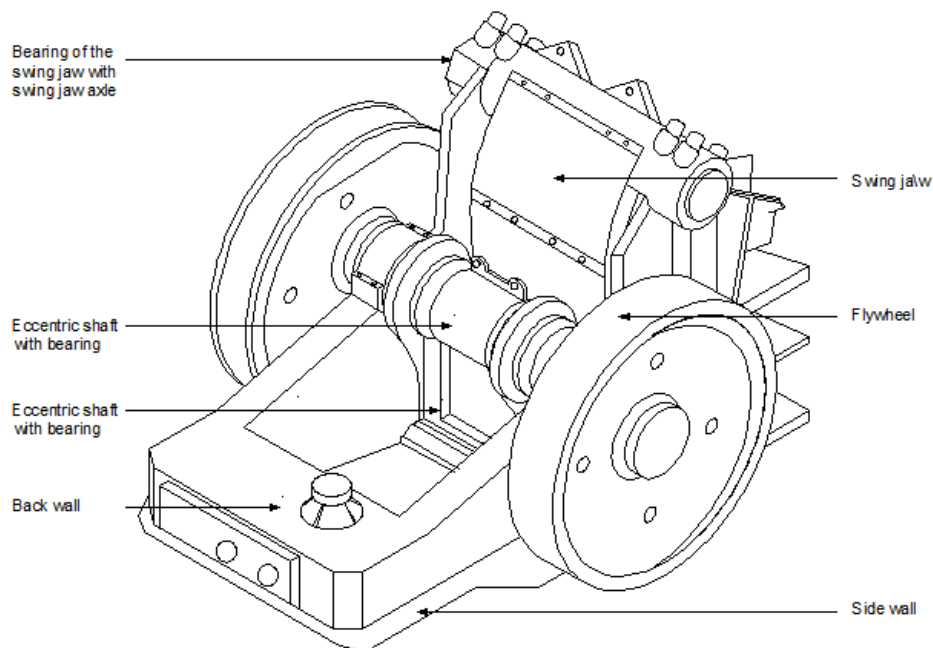


Figure 3: Double-toggle jaw crusher with its main components (Adapted from www.thyssenkrupp-industrial-solutions.com/backenbrecher_en.pdf)

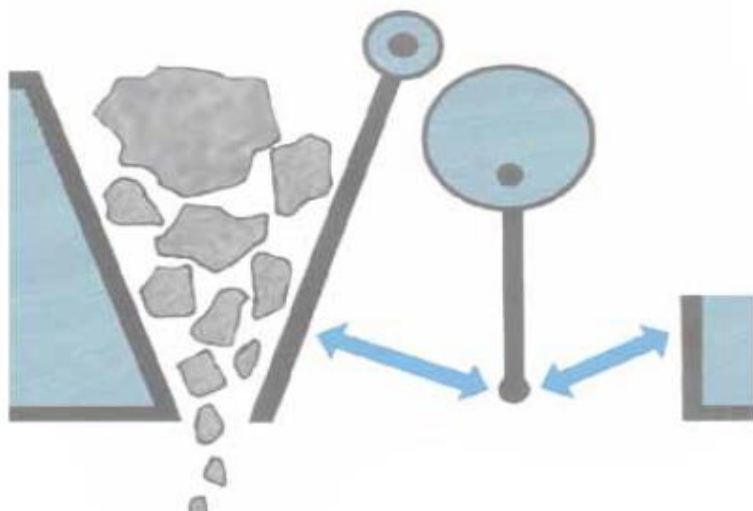


Figure 4. The Operating mechanism of a double-toggle jaw crusher with the toggle-lever system (www.thyssenkrupp-industrial-solutions.com/backenbrecher_en.pdf)

Table 1 shows the materials that the various parts of a jaw crusher are made. The selection of the materials is to ensure the jaw crusher last for long time even subject to very hard ore.

Table 1. Material for components of jaw crusher (Garnaik, 2010)

S/No	Component	Material / Function
1.	Body	Made from high quality steel plates and ribbed heavily in welded steel construction
2.	Swing jaw Plate	Manganese steel
3.	Fixed jaw plate	Manganese steel
4.	Pitman	Crushers have a light weight pitman having white-metal lining for bearing surface
5.	Toggle	Double toggles, for even the smallest size crushers give even distribution of load
6.	Flywheel	high grade cast iron
7.	Tension Rod	Pullback rods helps easy movement, reduces pressure on toggles and machine vibration
8.	Hinge plate	Strong hinge pin made from steel are used for crushing without rubbing
9.	Shaft and bearings	Massive rigid eccentric shafts made from steel along with roller bearing ensures smooth running.
10.	Diaphragm	Flexible diaphragm seals opening in oil chamber and protects components from dust

One of the trends in design solutions of crushers ensuring the crushing ratio of about 30 involves the application of the vibratory-impulse action to the material to be crushed. Crushers utilizing these effects are referred to as vibratory crushers. During the vibratory crushing the material to be disintegrated is subjected to the action of fast changing shearing forces, which leads to the material

being crushed either by applied impulses or by fatigue action, unlike conventional crushers where the structure of the material is damaged by the applied pressure (Wolny, 2013).

3.0 Dimensions and Operating Parameters

Feed characteristics are important parameters when designing or selecting a jaw crusher for a specific job (Napier-Munn *et al.*, 1999). The feed particle sizes of interest are those of the ore or rock that enters the crusher, the particle that can be nipped, particle that can fall through the chamber, and the particle that can fall through the chamber when the jaws are wide open (Donovan, 2003). The dimensions defined by these particle sizes include the gape which is the distance between the jaws at the feed opening, the Closed Side Set (CSS) which is the minimum opening between the jaws during the crushing cycle (minimum discharge aperture), the Open Side Set (OSS) which is the maximum discharge aperture, and the throw – the stroke of the swing jaw and the difference between OSS and CSS. Figure 5 shows a schematic of crusher dimensions (Donovan, 2003)

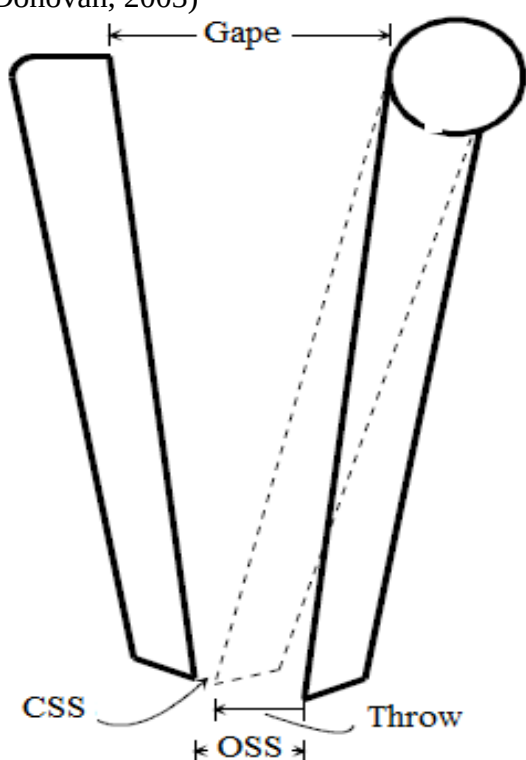


Figure 5. Schematic of crusher dimensions (Donovan, 2003)

The rating of jaw crushers is determined by the maximum dimensions of their feed opening, defined by the gape and the width of the plates. The feed openings of jaw crushers are known to vary from 250 mm by 500 mm to 1524 mm by 2032 mm. The largest particle size of the rock to be crushed should not be more than 80% of the rated gape (Weiss, 1985). The size of rectangular or square opening at the top of the jaws crusher defines the size the crusher. Therefore a 30 x 40 jaw crusher has an opening of 30" by 40". Primary jaw crushers generally come with square opening design while secondary jaw crushers have the rectangular opening design (Garnaik, 2010).

4.0 Selection of Crushers

The main factor that ensures effective control of the crushing process is proper selection of the crushing equipment. The principal factors that should be considered while selecting jaw crushers include (Donovan, 2003);

- a. The petrographic nature of the rock
- b. The abrasion index of the aggregate
- c. The mechanical strength of the rock
- d. The work index, or impact crushability, of the rock
- e. The brittleness of the rock
- f. The flakiness of the rock
- g. The feed size
- h. The desired product size (or reduction ratio)
- i. The throughput to be crushed (tons per hour)
- j. The capital cost
- k. The long-term operating costs.
- l. Project location, Operational considerations (e.g capacity per hour or day)
- m. Climatic conditions,
- n. Safety and environment,
- o. Life of mine/expansion plans and Maintenance requirements.

The petrographic nature of the rock and its resistance to crushing forces (characterized by strength, work index, etc.) are used first to determine what type of crusher is needed and then in part to determine the power requirement of the crusher. When a jaw crusher is selected outright, the physical properties of the rock can be used to determine the mechanical features of the jaw crusher, but for the most part they are used along with the feed size and desired throughput to determine the size of the jaw crusher and its power requirement.

Usually, the feed size range or the desired throughput dictates the selection of specific jaw crusher. The current trend is to go to the largest feed opening that is economically feasible in order to avoid as much secondary breakage as possible (Donovan, 2003). Once a crusher is selected it can be determined whether or not the crusher will provide the desired reduction ratio or product size distribution. As was noted previously the size of a jaw crusher's discharge opening controls the product size distribution. Thus, determining whether a specific jaw can produce the desired product is dependent upon whether or not that crusher can have its discharge opening set to the required dimension. Table 2 shows the capacities for various Nordberg series jaw crushers.

Screen analysis graphs (Figure 6) and capacity charts (Table 2) have greatly simplified the task of selecting a crusher for a specific task. This been applied extensively in the selection of Nordberg C Series jaw crushers (Donovan, 2003; Metso Minerals, 2003) where computer program has been developed to speed up the process. The program known as the *Bruno* program requires inputs such as rock type, feed gradation, and the desired crusher to simulate expected capacities and product size distribution curves (Metso Minerals, 2003). The use of crusher efficiency index number based on crusher manufacturer's published data and the crushability factor of the rock has been proposed by Wagner (1990) to assist in crusher selection. However, the use of information based on standard rock size such as those presented in graphs, charts, etc, may be inaccurate because of the

inhomogeneous nature of many rock types (Bearman, Barley and Hitchcock (1990). To carry out a selection that may be consider accurate will therefore necessitate the need to fully characterize the rock which may require a lot of laboratory tests and experiments.

Table 2. Capacities for various Nordberg C Series jaw crushers (After Metso Minerals, 2003, Donovan, 2003)

CSS* mm	C63 440 × 630	C100 760 × 1000	C110 850 × 1100	C140 1070 ×1400	C160 1200 ×1600	C200 1500 ×2000
40	40					
50	55					
60	65					
70	80	150	190			
80	95	170	210			
90	110	190	235			
100	120	215	255			
125		265	310	385		
150		315	370	455	455	520
175		370	425	520	595	760
200		420	480	590	675	855
225				655	750	945
250				725	825	1040
275					900	1130
300					980	1225

Capacities in metric tons per hour

*CSS stands for closed side set (CSS) - the minimum opening between the jaws during the crushing cycle (minimum discharge aperture)

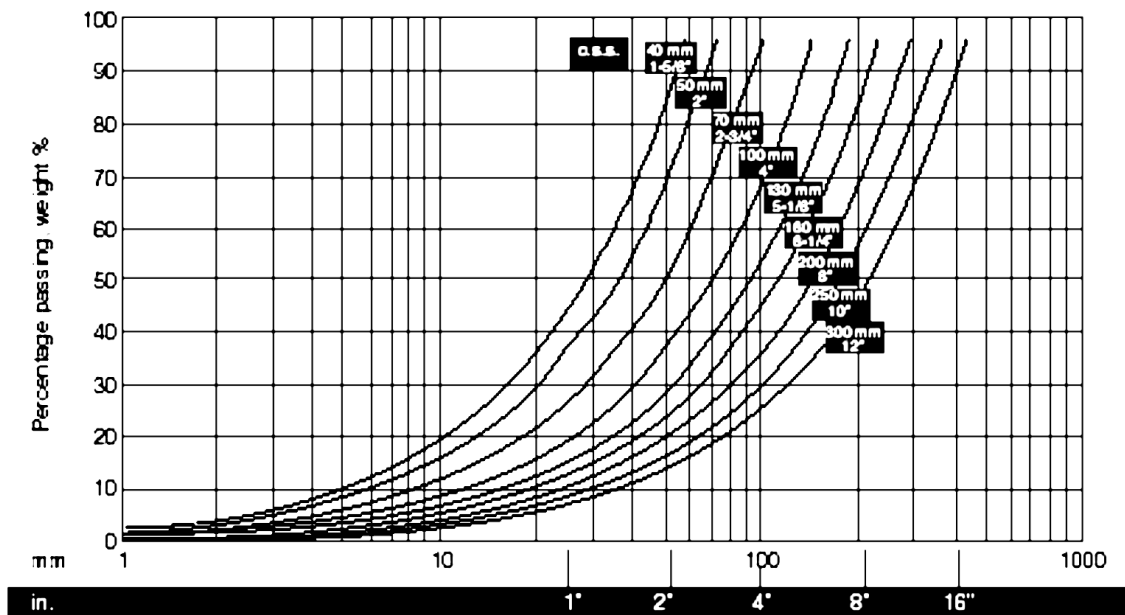


Figure 6. Product size distributions based on CSS (From Metso Minerals, 2003; Donovan, 2003)

For jaw crushers, a lot of considerations should be analysed such as (Peter mayo; <http://www.quarry.com.au>):

- a. The particle size of the should not be greater than 80% of the 'gape';
- b. The crusher's operating setting (close side setting);
- c. Maximum product particle diameter which normally is 1½ times the close side setting of the machine. This those not follow for slappy materials;
- d. It is normally considered that 50% - 60% of the crushed product will pass the close side setting;
- e. Jaw crushers reduction ratio are normally 6:1
- f. When selecting a jaw crusher as a primary crusher, it should be noted that its output should exceed the average capacity of the plant, as primary feed to the plant is generally of a cyclical nature, relying on trucks or loaders in most cases.
- g. For jaw crushers to operate optimally, all feed with particle size smaller than the closed side setting is removed to bypass the jaw crusher;
- h. Generally, maximum feed size is critical in the selection of jaw crushers and not required capacity. This is because putting too fine a feed into a jaw crusher normally leads to equipment failure because of overload.

A lot of dynamic forces are present during a crushing process. These forces and the load also affect the supporting structures and foundations. It is therefore very important to consider how these forces and load affect the crusher structures and the outcome incorporated in the design (Rusiński, 2013).

5.0 Energy and Size Reduction

Crushing theory is concerned with the relationship between the amount of energy put into a rock or particle of a known size and the reduced particle size after the comminution process has taken place. The amount of energy required to do this work is referred to as the 'Work Index' and it is an expression of the resistance that a material has to crushing (or grinding). It is a measurement of how hard the ore is. Therefore 'Work Index' can be defined as (<http://gravel.gravelwasher.com/metalliferous-mining-processing-resource-book-crushing.html>):

The amount of energy (work) required to break a rock from one size to another.

Each rock has a characteristic Work Index. For example, more energy is required to break quartz than sandstone so quartz will have a higher work index. Numerically the work index is known as the kilowatt-hours per tonne or kWh/t. A kilowatthour is the number of kilowatts consumed in an hour. For example, if you ran a 20kW motor for an hour, you would expend 20kWh. If you used that motor to do work to 2 tonnes of rock you would have used 20kWh/2 tonne, which is 10kWh/t. The higher the Work Index, the more work required to break the rock, hence the harder the rock. Whereas Work Index refers to the size fraction when rocks are reduced in size, Specific energy input refers to the amount of energy required to treat a certain tonnage. In simple terms if you crush 100t of rock from your feed size to your product size and it takes 180kWh, your specific energy input is 180kWh/100t = 1.8kWh/t.

The concepts of Work Index and Specific Energy make it clear that a certain (calculable) amount of energy is required to break a rock from one size to another. Experience has shown that to break

a rock from coarse and medium size should require less energy per unit mass than to break a rock from coarse to fine, i.e. the smaller you want to break a rock, from a given starting size, the more energy you require. Because we are talking about specific energy, i.e. energy per tonne of ore, it also follows that a given amount of energy spread out over a larger number of rocks will produce less breakage(<http://gravel.gravelwasher.com/metalliferous-mining-processing-resource-book-crushing.html>). This can be demonstrated by looking at the numbers:

For example: Say it takes 100kWh to break 50t of rock from 50mm to 10mm. Then the Specific Energy Input from 50mm to 10mm is $100\text{kWh}/50\text{t} = 2\text{kWh/t}$. If we then tried to use that same 100kWh to break 100t of rock, we would only be putting in $100\text{kWh}/100\text{t}$ or 1kWh/t .

Now, we know that to break a rock finer we need to increase the energy input, so a Specific Energy Input of 1kWh/t will produce a much coarser product than a Specific Energy Input of 2kWh/t .

6.0 Mechanisms of Particle Fracture

A size reduction process is analyzed by looking at the entire size distribution of its feed and product material. However, as each particle breaks as a result of the stresses applied to it and it alone, it is of value to look at single particle fracture. For a particle to fracture, a stress high enough to exceed the fracture strength of the particle is required. The manner in which the particle fractures depends on the nature of the particle, and on the manner in which the force is applied. There are three main generalized mechanisms of single particle fracture and they are(<http://gravel.gravelwasher.com/metalliferous-mining-processing-resource-book-crushing.html>).

6.1 Abrasion

Abrasion fracture occurs when insufficient energy is applied to cause significant fracture of the particle. Rather, localised stressing occurs and a small area is fractured to give a distribution of very fine particles. This type of breakage occurs predominantly in ball mills.

6.2 Cleavage

Fracture by cleavage occurs when the energy applied is just sufficient to fracture the particle. Only a few particles result and their size is comparatively close to the original particle. Typically, this situation occurs under conditions of slow compression where the fracture immediately relieves the loading on the particle. This is the main type of fracture found in jaw crushers.

6.3 Shatter

Fracture by shatter occurs when the applied energy is well in excess of that required for fracture; under these conditions many areas in the particle are overloaded and the result is a comparatively large number of particles with a wide size distribution. In practice these events do not occur in isolation, rather breakages involve a combination of fracture mechanisms.

7.0 Predicting Jaw Crusher Performance

Comminution concerns the breakage of brittle particles under conditions of applied compressive stress. The nature of the failure mechanisms is governed by material properties of the particulate material and by the nature of the stress field around and within individual particles. The response of the particulate material to the stress field is largely elastic but significant non-elastic behavior

occurs, particularly at the tips of growing cracks where large quantities of energy are dissipated when the criteria for fracture are met. The dissipation at the crack tip of the stored elastic energy in the particle turns out to be of critical interest in industrial comminution machines where energy efficiency is of major consequence because of its economic importance. Industrial comminution processes are typically inefficient in their use of energy in the sense that considerably more energy is consumed by the operating equipment than is actually required to break the particles. In spite of the importance of this observation, it has not been possible to calculate precisely how much energy is actually required (Umucu *et al.*, 2013).

Specific fracture energy of the particles is not the only fundamental property that is important. The particle strength also plays a significant role in determining the overall comminution properties of the material. A particle will be broken only if it is stressed beyond its strength which is determined by the intrinsic properties of the material, the presence of micro flaws which act as stress raisers when the particle is under load and the state of stress that is experienced by the particle (Umucu *et al.*, 2013).

The performance of jaw crusher is mainly determined by the kinetic characteristic of the liner during the crushing process. The practical kinetic characteristic of the liners which are located in certain domain of the coupler plane are computed and discussed by Jinxi *et al.*, (2007) and Shrivastava (2012).

The performance of a jaw crusher is most aptly defined using the product size, the capacity or throughput, and the power consumption. The main object of size reduction in the aggregate industry is to generate a well-shaped product within a specified size range with a minimum of fines. It is also desirable to maintain target production rates and to crush the rock material as efficiently as possible.

There are various methods for the prediction of crusher performance. As noted previously in the discussion of crusher selection, technical literature provided by crusher manufacturers can be used to determine capacities and product size. Adjustments can be made to account for physical variations in feed materials using the aforementioned laboratory tests. When the actual performance of a jaw crusher is difficult to determine or unclear to the manufacturer, laboratory tests using small-scale jaw crushers are sometimes conducted in order to predict the performance of full capacity machines. The normal procedure is to test crush a representative sample of the feed material and determine the product size distribution. Comparison of the product distribution and associated properties of the feed material to other materials is used to estimate overall crushing performance, power requirements, and to detect characteristics that may require special design considerations. These tests provide an opportunity to evaluate the effects of various crusher settings, speeds, feeding methods, and the physical characteristics of the material (Pennsylvania Crusher Corporation, 2003).

In addition to charts, graphs, and lab-scale crushing tests, jaw crushers can also be evaluated using mathematical modeling techniques. Mathematical techniques are usually derived from laboratory data and can be used to predict crusher performance and possibly improve selection and design procedures (Rimmer *et al.*, 1986). Models/equations are typically used to determine the product size, power draw, and capacity of crushers. The advantage of models is that they reduce complex

operations to a few numbers or parameters and can provide guidance to improved performance and decision making (Napier-Munn *et al.*, 1999).

7.1 Estimating capacity

The capacity of a jaw crusher is dependent upon the machine characteristics, the feed, and the nature of the rock material. The volumetric capacity of a jaw crusher can be estimated from the following equations (Sastri, 1994):

$$V = 60Nw (CSS + 0.5T) \left(\frac{DT}{G-(CSS+T)} \right) \text{ at low speeds} \quad (1)$$

$$V = 60Nw (CSS + 0.5T) \left(\frac{450g}{N^2} \right) \text{ at high speeds}$$

where N is the speed of the crusher in rpm

w is the width of the jaws in m

CSS is the closed side set in m

T is the throw in m

D is the vertical depth between the jaws in m

G is the crusher gape in m, and

g is gravitational acceleration in m/s^2 .

Equation 1 gives volumetric capacity under ideal conditions. At low crusher speeds the material falls down the chamber due to gravity flow, and the distance of the fall is dependent on the geometry of the machine. At higher speeds there is less time between strokes/revolutions and movement of material down the chamber is constricted and controlled by the speed of the machine (Hersam, 1923).

In order to determine the actual capacity of a jaw crusher, Equation (1) needs to be corrected by accounting for the feed size, the degree of compaction resulting from vibration, and the nature of the material. Equation (2) gives the estimated capacity using these parameters (Sastri, 1994):

$$V = 60Nw (CSS + 0.5T) \left(\frac{DT}{G-(CSS+T)} \right) K_1 K_2 K_3 \text{ at low speeds} \quad (2)$$

$$V = 60Nw (CSS + 0.5T) \left(\frac{450g}{N^2} \right) K_1 K_2 K_3 \text{ at high speeds}$$

with

$$K_1 = 0.85 - \left(\frac{F_{avg}}{G} \right)^{2.5} \quad (3)$$

$$K_2 = 1.92 \times 10^{\frac{-6.5T}{G}}$$

where, F_{avg} is the average feed size in m, and K_3 is a parameter related to the nature of the material. The parameter K_3 does not have a suggested value but is considered to increase with increasing hardness or toughness.

7.2 Product Size Distribution for Crushers

Specific fracture energy of the particles is not the only fundamental property that is important. The particle strength also plays a significant role in determining the overall comminution properties of the material. A particle will be broken only if it is stressed beyond its strength which is determined by the intrinsic properties of the material, the presence of micro flaws which act as stress raisers when the particle is under load and the state of stress that is experienced by the particle.

For impact breakage mechanism, many stakeholders are using the model of the percentage of broken product passing 1/10 the original particle size, t_{10} . The parameter t_a is defined as one tenth of t_{10} as follows (Napier-Munn *et al.*, 1996):

$$t_a = \frac{t_{10}}{10} \quad (4)$$

The abrasion breakage parameter, t_a , is determined by performing a tumbling test. A lower value of t_{10} (hence a lower t_a value) indicates that there is a lower percentage of material passing 1/10th the original particle size, or greater resistance to abrasion breakage.

Equation (5) relates t_{10} to E_{CS} which is specific energy of comminution (Napier-Munn *et al.*, 1996):

$$t_{10} = A (1 - e^{-bE_{cs}}) \quad (5)$$

where: t_{10} = percentage passing 1/10th of the initial mean size,

E_{cs} = specific comminution energy (kWh/kg)

A, b = ore impact breakage parameters.

The impact breakage parameters, namely A and b , are determined using a high energy impact breakage device called JK Drop Weight Tester. Specific energy of comminution, E_{cs} , in Equation (5) can be calculated according to experimental setup data, as follows (Napier-Munn *et al.*, 1996):

$$E_{cs} = \frac{0.0272M_d(h_i - h_f)}{\bar{m}} \quad (5a)$$

Where, M_d is drop weight mass (kg), m is the medium mass of per particle size class; (g), h_i is initial height of drop weight above the anvil (cm) and h_f is final height of drop weight above the anvil (cm).

It is noted that A is the maximum t_{10} value achieved. This is significant for higher energy breakage. The parameter b is related to the overall slope of t_{10} versus E_{cs} curve at the lower energies. A and b are interdependent, since the value of one will directly affect the other. Parameters A and b are related and usually are reported by $A.b$ as a single value to indicate the ore hardness in terms of impact breakage. The $A.b$ parameter is the slope of the t_{10} vs. E_{cs} curve at its origin and it is a measure of breakage of the ore at lower energy levels.

Napier-Munn *et al.*, indicate a possible correlation between the impact ($A.b$) and abrasion (t_a) parameters and the Bond ball mill work index given by the equations (Hasani *et al.*, 2009):

$$A \times b = -3.5W_i + 117 \quad (6)$$

$$t_a = 19.7W_i^{-1.34} \quad (7)$$

Narayanan (1983), Pauw and Mare (1988), Bourgeois (1993) and King (2002), King (2002) proposed breakage distribution model as (Soni *et al.*, 2013):

$$t_n = 1 - (1 - t_{10})^{\left[\left(\frac{9}{n-1}\right)^\alpha\right]} \quad (8)$$

Where, t_n = cumulative fraction passing from 1/nth of the feed size
 t_{10} = cumulative fraction passing from 1/10th of the feed size
 α = a material characteristic

The t_n versus t_{10} relationships can then be used to predict the product size distributions at different grind times (Sand and Subasinghe, 2004).

King (2002) proposed the relationship of t_{10} to the specific comminution energy by the equation:

$$t_{10} = t_{10(max)} \left[1 - e^{-\frac{\beta E_{CS}}{E_{50}}} \right] \quad (9)$$

where, E_{CS} = specific comminution energy (Kwh/ton).

The coefficient β and E_{50} (median impact energy at fracture) were considered as material characteristics by (Narayanan, 1987), (Napier-Munn, 1996) and (Weedon, 2001).

Umucu *et al.*, (2013) proposed a different size distribution relation for t_n values of crushing products obtained from different laboratory scale crushers (jaw, roller and hammer). The following equation was used to predict the cumulative percentage passing (t_n), depending on the Crushing Engine Power (CEP) and feed particle size (X) before crushing:

$$t_n = \left(\frac{56.12}{n^{1.168} CEP^{-1.274}} \right) X^{0.30} \quad (10)$$

The relationship has however not been tested on industrial crushers.

Narayanan (1986) used a novel procedure for estimation of breakage distribution functions of ores from the t-family of curves. In this method, the product size distribution can be represented by a family of curves using marker points on the size distribution defined as the percentage passing (t) at a fraction of the parent particle size. Thus, t_2 is the percentage passing an aperture of half the size of the parent particle size, t_4 is one quarter and t_{10} is one-tenth of parent particle size. Narayanan and Whiten (1988) have proposed empirical equations for relating the reference curve data t_{10} with the impact energy.

7.3 Product Size Models

Csoke *et al.*, (1996) developed an empirical method for determining the product size of “preliminary” crushers, i.e. jaw crushers and gyratory crushers. The product size resulting from breakage of the material larger than the CSS of the crusher can be modeled using the following function:

$$P(d) = \left(\frac{r}{r_{max}}\right)^m \quad (11)$$

$$\text{With } r = \frac{d_p}{Gap} \quad (12)$$

$$r_{max} = \frac{d_{pMAX}}{Gap} \quad (13)$$

where,

m is exponent describing the product size distribution

d_p is the particle size

d_{pMAX} is the largest size in the product (OSS), and

Gap is the CSS (CSS is closed side set (CSS) - the minimum opening between the jaws during the crushing cycle (minimum discharge aperture)) of the crusher.

Based on operating data found in the literature, Equation (11) fits the product size of a jaw crusher with r_{max} equal to 1.223 and m of 0.842.

King (2001) outlined a model for the prediction of product size based on data from a crusher manufacturer. The main concept is that the size distribution of the product is characterized by the particle size relative to the OSS and by the product type, P_t , which is the fraction of the product smaller than the OSS. P_t is related to the nature of the material and is distinguished by the work index and a qualitative description of the material. The size distribution given by this model is determined from the following equations:

$$P(d) = 1 - e^{\left(-\left[\frac{r}{K_u}\right]^{1.5}\right)} \quad \text{for } r > 0.5 \quad (14)$$

$$P(d) = 1 - e^{\left(-\left[\frac{r}{K_l}\right]^{0.85}\right)} \quad \text{for } r < 0.5$$

With

$$r_i = \frac{D_i}{OSS} \quad (15)$$

$$K_u = \left[\ln \left(\frac{1}{1-P_1} \right) \right]^{-0.67} \quad (16)$$

$$K_1 = 0.5 \left[\ln \left(\frac{1}{1-P_b} \right) \right]^{-1.18} \quad (17)$$

$$P_b = 1 - e^{\left(-\left(\frac{0.5}{K_u} \right)^{1.5} \right)} \quad (18)$$

Equation (14) should only be used when the size at which half of the feed material passes is greater than the OSS.

Other product size models of note is the one developed by Whiten in 1972 (Donovan, 2003).

First the classification and breakage could be illustrated by Figure 7.

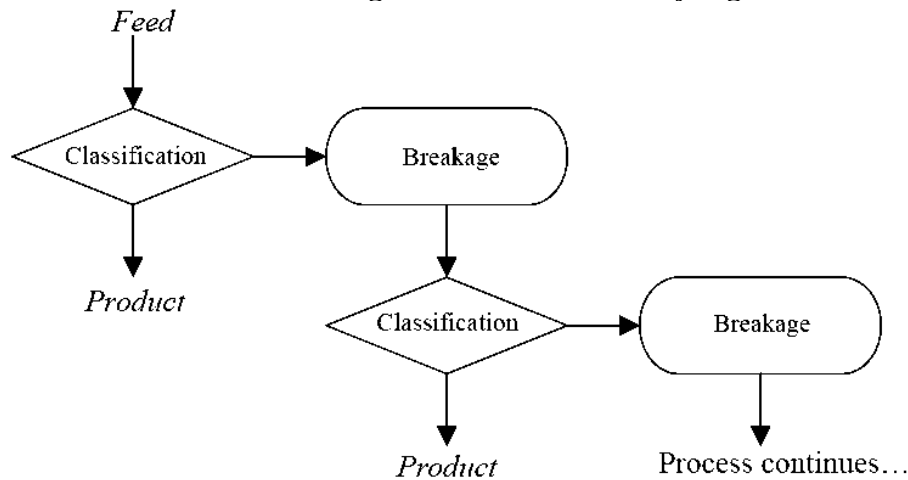


Figure 7. Flowchart of classification and breakage process (Donovan, 2003)

If the classification and breakage of particles in a jaw crusher is considered as a closed circuit process, Figure 7 can be represented by Figure 8, a representation of the Whiten crusher model (Donovan, 2003).

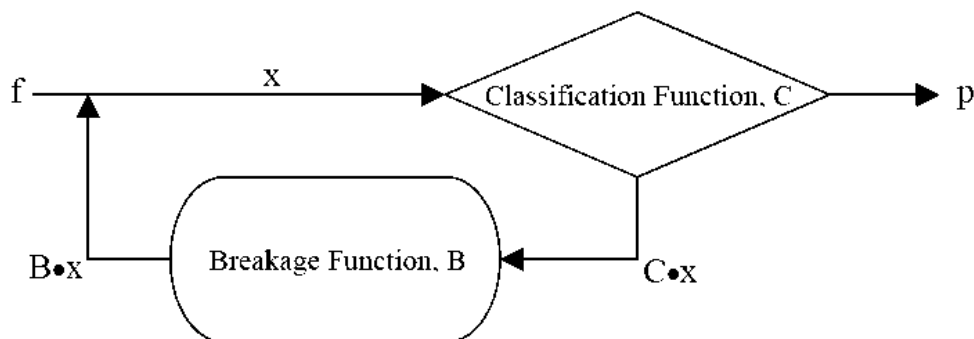


Figure 8. Whiten crusher model (Donovan, 2003)

The Mass balance equations written about each node of Figure 8 describe the repetitive process of classification and breakage and are expressed as (Kojovic *et al.*, 1997, Donovan, 2003):

$$X = f + Bx \quad (19)$$

$$X = p + Cx \quad (20)$$

where, x is a vector representing the amount in each size fraction entering the crusher

f is the feed size distribution vector

p is the product size distribution vector

C is the classification (diagonal) matrix

B is the breakage distribution (lower triangular) matrix.

The classification matrix describes the proportion of particles in each size interval entering the crushing zone. The breakage distribution matrix gives the relative distribution of each size fraction after a breakage event. Equations (19) and (20) can be combined resulting in the Whiten crusher model equation (Whiten, 1972):

$$P = (I - C) \cdot (I - BC)^{-1} \cdot f \quad (21)$$

where, I is the unit matrix. Equation (21) can be used to determine the product size of a jaw crusher given the feed size, classification function, and the breakage function (Donovan, 2003).

7.4 Power Models for Jaw Crushers

The classical comminution theories, which were derived by Rittinger, Kick and Bond (Lee, 2012) respectively, aim to describe the relation between energy and size reduction for a given feed size. The three theories, which are usually referred to as the first, second and third theory of comminution, are formulated in Equations (22) to (24) (Asbjörnsson, 2013; Lee, 2012).

$$E_{Rittinger} = C_{Ritt} \left(\frac{1}{p_{80}} - \frac{1}{f_{80}} \right) \quad (22)$$

Where $E_{Rittinger}$ is the energy input, C_{Ritt} is a material constant, p_{80} is the size which 80 % of the product passes, and f_{80} is the size which 80 % of the feed passes (Asbjörnsson, 2013; Lee, 2012).

$$E_{Kick} = -C_{Kick} \ln \left(\frac{p_{80}}{f_{80}} \right) \quad (23)$$

where E_{Kick} is the energy input and C_{Kick} is a material constant.

Rittinger's and Kick's laws lack precision in describing the shape of a size distribution curve as only a single point is used (Lee, 2012).

The third and final classical relationship was proposed by Bond in 1952. According to his theory, the energy input is proportional to the length of the produced crack tip and is equal to the difference in energy represented by the product and the feed, and is expressed as (Lee, 2012):

$$E_{Bond} = C_{Bond} \left(\frac{1}{\sqrt{p_{80}}} - \frac{1}{\sqrt{f_{80}}} \right) = W_1 \left(\frac{10}{\sqrt{p_{80}}} - \frac{10}{\sqrt{f_{80}}} \right) \quad (24)$$

where p and f are in microns. W_1 is the Bond's *work index* which expresses the resistance of a material to crushing or grinding. This constant is a function of material properties and the efficiency of the crusher. A lower efficiency in a crusher will give a higher work index (Asbjörnsson, 2013; Lee, 2012).

The above classical laws of comminution have not been found to accurate in many cases and others have worked to improve them with varying degree of success. Hukki in 1975 (Lee, 2012) suggested that all three relationships were in fact applicable but in different and relatively narrow size ranges (Will and Napier-Munn, 2005; Morrell (2004). Based on Hukki's work, Morrell (2004) later successfully proposed a modification of Bond's equation. Unfortunately, as pointed out by Lindqvist (2008), Morrell's modified energy model still lacks precision due to the sole use of f_{80} and P_{80} . Egbe *et al.*, (2013) presented procedures for determining the Bond's work index of some Nigerian ores. An alternative approach to calculate the energy cost to produce a given product size distribution from a given feed size distribution was suggested by Lindqvist (2008) and Lee (2012).

However, there is generally little confidence in the model results and it is also suspected that all these underlying ideas are incorrect and that the energy-size reduction relationship does not suitably define the process of size reduction (Napier-Munn, 1996, Lynch, 1977). Bond's "law" is an empirical law that only fits experimental data over a limited range of variables and only in certain cases (Choi, 1982). It can be corrected for other operating conditions but even with this correction it has been shown that the power consumption of a jaw crusher can be as much as 240% greater than the expected consumption based on Bond's equation (Eloranta, 1997). Rose and English have suggested a relationship for power consumption based on Bond's work index but take into account the material density and machine characteristics, i.e., gape, throw, CSS, etc. (Lowrison, 1974).

A more recent approach for predicting power consumption in crushers was presented Andersen and Napier-Munn (1988) and Morrell *et al.*, 1992. In this case, they used using a single regression equation relating the power actually drawn by the industrial machine in producing a particular product size distribution, to the calculated power required to achieve the same size reduction in a laboratory test (Sabir and Forlin, 2002, Donovan, 2003).

$$P_c = AP_p + P_n \quad (25)$$

where P_c is the power drawn by the crusher under load in kW and P_p is the pendulum power in kW expressed as

$$P_p = \sum_{i=1}^j E_{cs} t_{10i} C_i x_i \quad (26)$$

P_n is the power drawn by the crusher under no load in kW.

This power model provides a means of comparing the power efficiency of different patterns and types of crushers, in terms of efficiency constant A, and has been found to provide satisfactory

predictions for several crusher types. However, as the regression coefficients are not valid for all crushers, the tests must be repeated for some crusher types (Sabir and Forlin, 2002).

Today the energy-size relation in the comminution process is usually presented in the form of the differential equation proposed by Walker *et al.*, (1937) and revised by Hukki (1962). This is represented by Equation (27) (Asbjörnsson, 2013):

$$dE = -C \frac{dx}{x^{f(x)}} \quad (27)$$

The most common mathematical model used today for expressing the crusher performance is the empirical Selection-Breakage model proposed by Whiten (1972). This model is however empirically fitted to data from drop weight tests, which is more suitable for mills than compressive crushers and does not predict capacity.

More detailed analytical models based on the mechanical properties of the crusher have been proposed by Evertsson (2000) and Briggs (1997). These models can provide more accurate information from the process but require detailed information about crusher geometry, breakage behaviour and require considerable simulation time (Asbjörnsson, 2013).

7.5 Size Reduction Relationships

The general size reduction equation is as follows (Morrell, 2004):

$$W_i = M_i 4 \left(x_2^{f(x_2)} - x_1^{f(x_1)} \right) \quad (28)$$

Where:

W_i = Specific comminution (kWh/tonne);

M_i = Work index related to the breakage property of an ore (kWh/tonne);

x_2 = 80% passing size for the product (microns)

x_1 = 80% passing size for the feed (microns)

$f(x_j)$ is calculated from the relationship: ((Morrell, 2006):

$$f(x_j) = -(0.295 + x_j \times 10^{-6}) \quad (29)$$

For grinding from the product of the final stage of crushing to a P_{80} of 750 microns (coarse particles) the index is labeled M_{ia} and for size reduction from 750 microns to the final product P_{80} normally reached by conventional ball mills (fine particles) it is labeled M_{ib} . For conventional crushing, M_{ic} is used and for High Pressure Grinding Rolls, (HPGRs), M_{ih} is used.

For coarse particle grinding in tumbling mills Equation (28) is written as:

$$W_a = K_1 M_{ia} 4 \left(x_2^{f(x_2)} - x_1^{f(x_1)} \right) \quad (30)$$

where $K_1 = 1.0$ for all circuits that do not contain a recycle pebble crusher and 0.95 where circuits do have a pebble crusher; $x_1 = P_{80}$ in microns of the product of the last stage of crushing before grinding; $x_2 = 750$ microns and M_{ia} = Coarse ore work index and is provided directly by Semi-

autogenous Mill Comminution (SMC) Test.

For fine particle grinding Equation (28) is written as:

$$W_b = M_{ib} 4 \left(x_3^{f(x_3)} - x_2^{f(x_2)} \right) \quad (31)$$

where $x_2 = 750$ microns; $x_3 = P_{80}$ of final grind in microns; M_{ib} is provided by data from the standard Bond ball work index test using the following equation (Morrell, 2006):

$$M_{ib} = \frac{18.18}{P_1^{0.295} (Gbp) (p_{80}^{f(p_{80})} - f_{80}^{f(f_{80})})} \quad (32)$$

where M_{ib} = fine ore work index (kWh/tonne); P_1 = closing screen size in microns; Gbp = net grams of screen undersize per mill; P_{80} = 80% passing size of the product in microns and f_{80} = 80% passing size of the feed in microns. Note that the Bond ball work index test should be carried out with a closing screen size which gives a final product P_{80} similar to that intended for the full scale circuit.

Equation (28) for conventional crushers is written as:

$$W_c = S_c K_2 M_{ic} 4 \left(x_2^{f(x_2)} - x_1^{f(x_1)} \right) \quad (33)$$

Where S_c = coarse ore hardness parameter which is used in primary and secondary crushing situations. It is defined by Equation (34) with K_s set to 55; $K_2 = 1.0$ for all crushers operating in closed circuit with a classifying screen; $x_1 = P_{80}$ in microns of the circuit feed; $x_2 = P_{80}$ in microns of the circuit product and M_{ic} = Crushing ore work index and is provided directly by SMC Test. If the crusher is in open circuit, eg pebble crusher in an Autogenous/Semi Autogenous, AG/SAG circuit, K_2 takes the value of 1.19.

The coarse ore hardness parameter (S) makes allowance for the decrease in ore hardness that becomes significant in relatively coarse crushing applications such as primary and secondary cone/gyratory circuits. In tertiary and pebble crushing circuits it is normally not necessary and takes the value of unity. In full scale HPGR circuits where feed sizes tend to be higher than used in laboratory and pilot scale machines the parameter has also been found to improve predictive accuracy. The parameter is defined by Equation (34);

$$S = K_s (x_1 x_2)^{-0.2} \quad (34)$$

Where K_s = machine-specific constant that takes the value of 55 for conventional crushers and 35 in the case of HPGRs; $x_1 = P_{80}$ in microns of the circuit feed and $x_2 = P_{80}$ in microns of the circuit product.

High Pressure Grinding Rolls (HPGR)

Equation (28) for HPGR's crushers is written as:

$$W_h = S_h K_3 M_{ih} 4 \left(x_2^{f(x_2)} - x_1^{f(x_1)} \right) \quad (35)$$

Where S_h = coarse ore harness parameter as defined by Equation 2.34 and with K_s set to 35 $K_3 = 1.0$ for all HPGRs operating in closed circuit with a classifying screen. If the HPGR is in open circuit, K_3 takes the value of 1.19. Also $x_1 = P_{80}$ in microns of the circuit feed; $x_2 = P_{80}$ in microns of the circuit product; M_{ih} = HPGR ore work index and is provided directly by SMC Test.

Specific Energy Correction for Size Distribution (Ws)

It should be noted that the feed and product size distributions are usually different (an assumption that was made in arriving in the above specific energy requirement) and so allowances for additional specific energy. However, for the purposes of this approach it has been assumed that the additional specific energy for ball milling is the same as the difference in specific energy between open and closed crushing to reach the nominated ball mill feed size. This assumes that a crusher would provide this energy. However, in this situation the ball mill has to supply this energy and it has a different (higher) work index than the crusher (that is, the ball mill is less energy efficient than a crusher and has to input more energy to do the same amount of size reduction). Hence from Equation (33), to crush to the ball mill circuit feed size (x_2) in open circuit requires specific energy equivalent to:

$$W_c = 1.19 * M_{ic} 4 \left(x_2^{f(x_2)} - x_1^{f(x_1)} \right) \quad (36)$$

closed circuit crushing the specific energy is:

$$W_c = 1 * M_{ic} 4 \left(x_2^{f(x_2)} - x_1^{f(x_1)} \right) \quad (37)$$

The difference between the two (Equation (36) and Equation (37)) has to be provided by the milling circuit with an allowance for the fact that the ball mill, with its lower energy efficiency, has to provide it and not the crusher. Thus W_s (specific energy correction for size distribution) is represented by:

$$W_s = 0.19 * M_{ia} 4 \left(x_2^{f(x_2)} - x_1^{f(x_1)} \right) \quad (38)$$

It should be noted that in Equation (38), M_{ic} has been replaced with M_{ia} , the coarse particle tumbling mill grinding work index.

7.6 Sizes of aggregate produced by jaw crushers

Even though the setting of the discharge opening of a crusher will determine the maximum size of aggregate produced, the aggregate sizes will range from slightly greater than the crusher setting to fine dust. For any given setting for jaw or roll crushers, approximately 15% of the total amount passing through the crusher will be larger than the setting. If the opening of the screen which receives the output from such crusher are the same size as the crusher setting, 15% of the output will not pass through the screen.

Figure 9 (Assakkaf, 2003) provides the percentage of material passing or retained on screens having the size opening indicated for jaw and roll crushers. To read the chart:

- Select the vertical line corresponding to the crusher setting
- Then go down this line to the number which indicate the size of screen opening
- From the size of the screen opening, proceed horizontally to the left to determine the percent of material passing through the screen or the right to determine the percentage of material retained on the screen.

Example 1 (Assakkaf, 2003)

A jaw crusher with a closed setting of 3½ in produces 60 tons per hour of crushed stone. Determine the amount of stone produced in tons per hour within the following size range: in excess of 1 in; between 1 and ½ in; between ½ and ¼ in.

It can be seen from Figure 9 that the amount retained on a 1-in screen is 71% of 60, which is 42.6 tons per hr. Also, details for each of the size range have been determined and calculations made as shown on Table 3:

Table 3. Sizes of aggregate produced by jaw crushers

Size Range (in)	% Passing Screen	%in Size Range	Crusher's Total Output (ton/hr)	Amount Produced in Size Range (ton/hr)
Over 1	100 – 29	71	60	42.6
1 – ½	29 – 17	12	60	7.2
½ – ¼	17 – 11	6	60	3.6
¼ - 0	11 – 0	11	60	6.6
Total		100%		60.0 tph

Examples 2 (Adapted from; Using The Smc Test® to Predict Comminution Circuit Performance (http://www.smctest.com/documents/Using_the_SMC_Test.pdf))

Calculate the overall specific energy to mill a rock in a primary crusher from a P₈₀ of 110 mm to a product with a P₈₀ of 116 microns for the following three circuit SAB mill crusher, SABC, HPGR/ball mill and Conventional crushing/ball mill. SMC Test and Bond ball work index test on a similar sample has generated the following details: SMC Test: $M_{ia} = 20.2$ kWh/t, $M_{ic} = 8.2$ kWh/t, $M_{ih} = 14.8$ kWh/t Bond test carried out with a 160-micron closing screen: $M_{ib} = 19.7$ kWh/t.

Solution

SABC Circuit

Coarse particle tumbling mill specific energy

Since it has a pebble crusher in the circuit, $k_1 = 0.95$. Also, 110mm = 110,000 microns. Combining Equations (29) and (30):

$$W_a = 0.95 * 20.2 * 4 * (750^{-(0.295+750 \times 10^{-6})} - 110,000^{-(0.295+110,000 \times 10^{-6})})$$

$$= 10.14 \text{ kWh/t}$$

Fine particle tumbling mill

specific energy

Combining Equations (29) and (31):

$$W_b = 19.7 * 4 * (116^{-(0.295+116 \times 10^{-6})} - 750^{-(0.295+750 \times 10^{-6})})$$

$$= 8.27\text{kWh/t}$$

Pebble crusher specific energy

The assumption for this circuit is that the feed to the pebble crusher has a P₈₀ of 60mm. Normally, this value is calculated by assuming that it is 0.75 of the nominal pebble port apertures which is 80mm in this case. The setting of the pebble crusher produces a product with P₈₀ of 14mm. The feed rate of the pebble crusher feed rate is estimated to be 30% of new feed tph. Combining Equations (29) and (33):

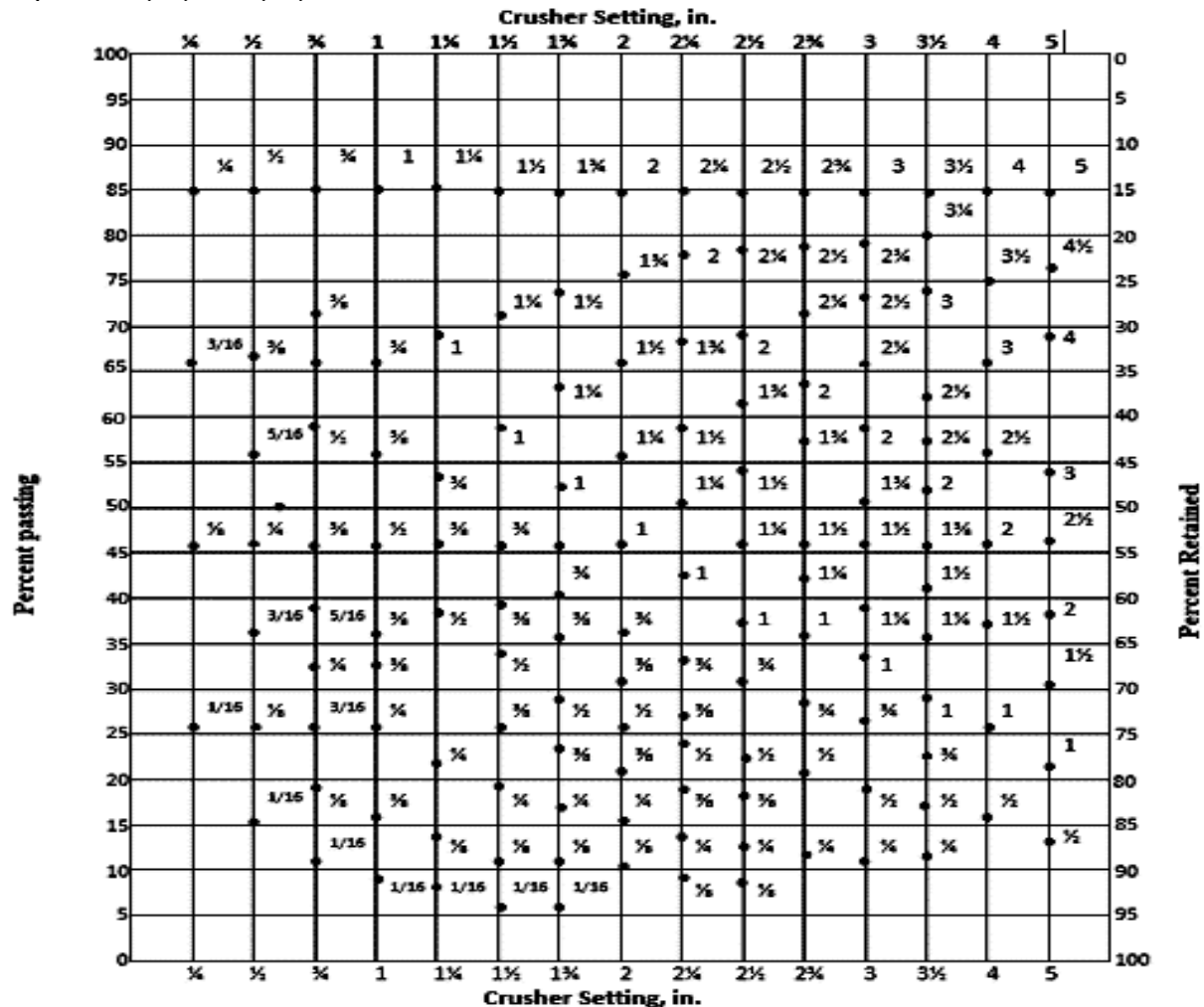


Figure 9. Analysis of the size of aggregate by jaw crushers

$$W_c = 1.19 * 8.2 * 4 * (14000^{-(0.295+14000 \times 10^{-6})} - 60000^{-(0.295+60000 \times 10^{-6})})$$

= 1.26 kWh/t when expressed in terms of the crusher feed rate

= 1.26* 0.30 kWh/t when expressed in terms of the SABC circuit new feed rate

= 0.38kWh/t of SAG mill circuit new feed

Total net comminution specific energy:

$$W_T = 10.14+ 8.27+ 0.38 \text{ kWh/t}$$

$$= 18.79\text{kWh/t}$$

HPGR/Ball Milling Circuit

The assumption in the circuit is that the primary crusher product is reduced to a HPGR circuit feed P₈₀ of 38 mm by a closed-circuit secondary crushing. The HPGR is also considered to be in the closed circuit and reduces the 38 mm feed to a circuit product P₈₀ of 6 mm. after this, the product is feed to a closed-circuit ball mill which takes the crushing down to a P₈₀ of 116 microns.

Secondary crushing specific energy

Combining Equations (29), (33) and (34):

$$W_c = 1 * 55 * (38000 * 110000)^{-0.2} * 8.2 * 4 \\ * (38000^{-(0.295+38000 \times 10^{-6})} - 110,000^{-(0.295+110,000 \times 10^{-6})}) \\ = 0.446\text{kWh/t}$$

HPGR specific energy

Combining Equations (29) and (34):

$$W_c = 1 * 35 * (6000 * 38000)^{-0.2} 14.8 * 4 \\ * (6000^{-(0.295+6000 \times 10^{-6})} - 38000^{-(0.295+38000 \times 10^{-6})}) \\ = 1.90\text{kWh/t}$$

Coarse particle tumbling mill specific energy

Combining Equations (29) and (30):

$$W_a = 1 * 20.2 * 4 * (750^{-(0.295+750 \times 10^{-6})} - 6000^{-(0.295+6000 \times 10^{-6})}) \\ = 5.50\text{kWh/t}$$

Fine particle tumbling mill specific energy

Combining Equations (29) and (31):

$$W_b = 19.7 * 4 * (116^{-(0.295+116 \times 10^{-6})} - 750^{-(0.295+750 \times 10^{-6})})$$

$$= 8.25 \text{ kWh/t}$$

Total net comminution specific energy:

$$W_T = 0.45 + 1.90 + 5.50 + 8.25 \text{ kWh/t}$$

$$= 16.1 \text{ kWh/t}$$

Conventional Crushing/Ball Milling Circuit

The assumption in this circuit is that the primary crusher product is reduced in size to P₈₀ of 6.8 mm via a secondary/tertiary crushing circuit (closed) which is then fed to a closed-circuit ball mill which reduces the product to a P₈₀ of 116 microns.

Secondary/tertiary crushing specific energy

Combining Equations (29) and (33):

$$W_c = 1 * 8.2 * 4 * (6800^{-(0.295+6800 \times 10^{-6})} - 110,000^{-(0.295+110,000 \times 10^{-6})})$$

$$= 1.99 \text{ kWh/t}$$

Coarse particle tumbling mill specific energy

Combining Equations (29) and (30):

$$W_a = 1 * 20.2 * 4 * (750^{-(0.295+750 \times 10^{-6})} - 6800^{-(0.295+6800 \times 10^{-6})})$$

$$= 5.76 \text{ kWh/t}$$

Fine particle tumbling mill specific energy

Combining Equations (29) and (31):

$$W_b = 19.7 * 4 * (116^{-(0.295+116 \times 10^{-6})} - 750^{-(0.295+750 \times 10^{-6})})$$

$$= 8.27 \text{ kWh/t}$$

Size distribution correction

$$W_s = 0.19 * 20.2 * 4 * (6800^{-(0.295+6800 \times 10^{-6})} - 110,000^{-(0.295+110,000 \times 10^{-6})})$$

$$= 0.93 \text{ kWh/t}$$

Total net comminution specific energy:

$$W_T = 1.99 + 5.76 + 8.27 + 0.93 \text{ kWh/t} = 16.95 \text{ kWh/t}$$

Worked Example 3: Feed Size

For a roll crusher, the diameter of the roll is directly proportional to the maximum particle diameter of the rock or ore that can be fed into it. This therefore means that very large diameter particles cannot be gripped by the rollers, and if they are too small, they simply fall through with size reduction.

For roll crusher that has an angle of nip, B , of 18.86° , the maximum particle size that can be crushed by the roll crusher is calculated as follows:

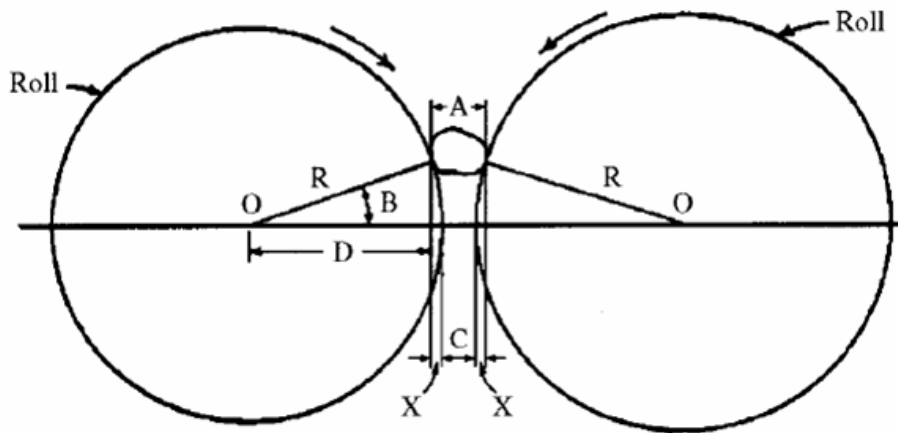


Figure 9. Crushing Rocks between two rolls

Let

R = radius of rolls

B = angle of nip

$D = R \cos B = R \cos (18.86) = 0.9463 R$

A = maximum particle size of the feed

C = roll setting which also equals the size of finished product

$X = R - D = R - 0.9463R = 0.0537R$

$A = 2X + C = 2(0.0537R) + C = 0.1074R + C$

$\therefore$ Maximum size Feed (A) = $0.1074R + C$

Therefore, if we are to estimate the maximum particle size of rock or ore that can be crushed by a roll crusher with a roll of diameter, 80 cm, and a roll setting of 2.0 cm, then:

Maximum particle size of the feed (A) = $0.1074R + C$

$A = 0.1074(80) + 2 = \underline{10.59cm}$

8.0 Conclusion

This paper reviews the selection process as well as the theoretical performance evaluation procedures for jaw crushers. It is however important to note that Crushers encounter considerable difficulties concerning external loads acting on their mechanical joints which make it almost impracticable to use theoretical calculations for design and performance evaluation. It is therefore necessary to use experimental studies to verify any theoretical methods used for these purposes.

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